Stat 561: Advanced Theory of Statistical Inference, Section 1, Spring Semester 2018

Time and Place: MWF 10:10 am – 11:00 pm, 219 Thomas.

Instructor information:

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2. Office: Thomas 410
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4. Office hours: MW 4:00 pm - 5:00 pm.

Texts and references:

1. *Mathematical Statistics: A Decision Theoretic Approach*, Thomas Ferguson.
2. *Probability and Measure*. Second Edition. Patrick Billingsley. 1986 John Wiley & Sons.
3. *Theory of Point Estimation*. Second Edition. Lehmann and Casella. Springer.
4. *Testing Statistical Hypothesis*. Second Edition. Lehmann. Springer.
5. *Statistical Decision Theory and Bayesian Analysis*. James Berger. Springer.
6. *Bayesian Inference*. Anthony O'Hagan. Cambridge University Press.

Examinations: There will be a midterm and a final exam. They will be closed book and in class. You may bring a page of prepared notes for each exam.

Assignments: Homework will be assigned and graded every two weeks.

Evaluation: Midterm 30 %. Final 45%. Homework 25%.

Course Coverage:

1 Bayesian Inference

1.1 Basic concepts: Prior; Likelihood; Posterior; Bayesian sufficiency.

1.2 Calculation of posterior: Conjugate families; Conditional conjugacy; improper priors; invariance of priors.

1.3 Bayesian decision theory: risk; posterior expected loss; Bayes risk; Bayes rule; Generalized Bayes rule.

1.4 Bayesian inference: Bayes rules for estimation, hypothesis test, and classification. Bayes rules and unbiasedness.

1.5 Stein's phenomenon: Stein's lemma, inadmissibility of MLE, James-Stein estimator.

1.6 Empirical Bayes Analysis: large number of parameters; factorization assumption; estimation of hyperparameters. James-Stein estimator as empirical Bayes estimator.

1.7 Classification.

2 Finite-sample theory of hypotheses test

2.1 Simple vs simple test: Two types of errors; The Neyman-Pearson Lemma.

2.2 One sided test: Size and power; Monotone likelihood ratio; UMP test.

2.3 Two sided test: One parameter exponential family; Generalized Neyman-Pearson Lemma; Uniform maximizationof power under two constraints; Uniform most powerful test (UMPU) when the null set is an interval; UMPU test when the null set is a point; UMP test when alternative set is an interval.

2.4 Test of one parameter in the presence of several: Exponential families with several parameters; Similar test; Uniformly most powerful similar test (UMPS); Sufficiency, bounded completeness, and Neyman structure; UMPU test with nuisance parameters; One sided and two sided cases; Testing linear functions for of natural parameters.

2.5 Constructing UMPU test in the presence of nuisance parameters: invariant family under a group of transformations; ancillary; Constructing UMPU test using Basu's theorem.

2.6 Converting an optimal test to an optimal confidence set.

3 Asymptotic theory of hypothesis test

3.1 von Mises expansion of statistical functionals

3.2 Asymptotically linear and quadratic statistics

3.3 Asymptotic quadratic form (AQF) tests

3.4 Important AQF tests: Wilk's test; Rao's test; Wald's test; Neyman-Rao's test; Lagrangian Multiplier test

3.5 Asymptotic null distribution of AQF tests.
